Name_____

ECE 179d

Final Exam – Fall 2019

- Extra pages are provided at back of exam. **Turn in all work!**
- Please write ONLY ON THE FRONT sides of exam pages.
- You are allowed two (2) single-sided sheets of notes (or one double-sided sheet)
- No calculators or other computing devices are allowed.

Good Luck!

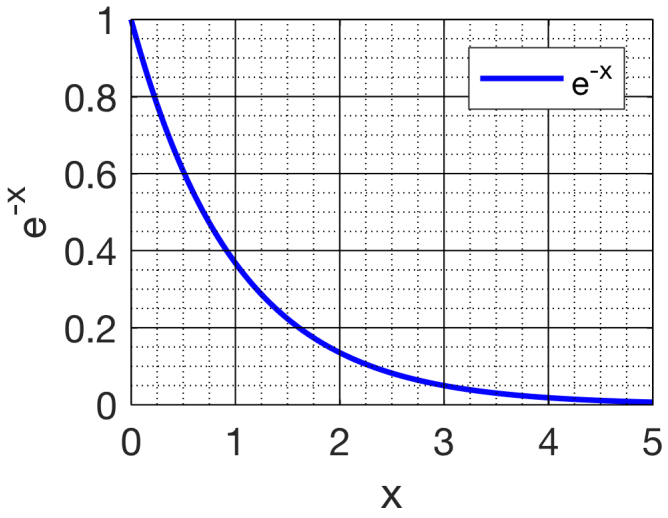
P1 _____

P2 _____

P3 _____

P4 _____

_____ Total Score



Problem	(a)	(b)	(c)	(d)	Total Pts
1	6	8	8	6	28
2	15	6	6		27
3	20				20
4	10	9	6		25

<u>Problem 1</u> – State space equations and linearization

(a) Determine the A and B matrices that correspond to the following dynamic system:

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -20x_1 - 8x_2 + 4u_1$$

where $X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ and $U = [u_1]$.

(b) Now, assume we use "state feedback" of the following form: $U = -KX$. Solve for K such that the closed-loop poles of the system will both be at $s = -10$. (In other words, we will have two closed-loop poles with $\omega_n = 10$ (rad/s) and $\zeta = 1$.) Be careful to get the correct "size" of K, to match your equations in part (a).

(c) Now, linearize a new system, as defined below, to generate A and B matrices, ONLY.

$\dot{y} = 2y + 2z^2 + 2u$

$\dot{z} = -5y - 2z$

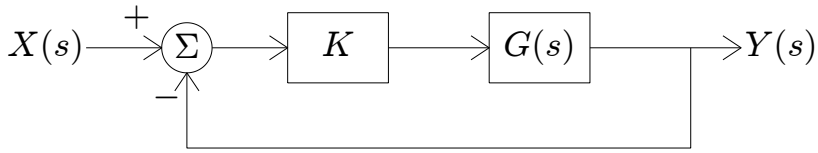
Specifically, linearize the system near $y^* = -0.8$, $z^* = 2$, $u^* = 3.2$ (At these values, $\dot{y} = \dot{z} = 0$). Define a linearized model so that $U = [u - u^*]$, and $X = \begin{bmatrix} y - y^* \\ z - z^* \end{bmatrix}$. Note: you are <u>ONLY</u> asked to find the A and B matrices here! (Hint: there is only one "nonlinear" term… so most of the terms require no special treatment.)

(d) What are the open-loop poles of the linearized system from part (c)? (If you write the polynomial in s that determines the poles, you will get most of the credit just for doing this… If you did not solve part (a), then define A and B matrices with open variables (a, b, c, …), to show you know what size they should be, and to be able to proceed with part (d), here.)

See Bode plot on next page!

Problem 2 – Loop shaping and stability margins

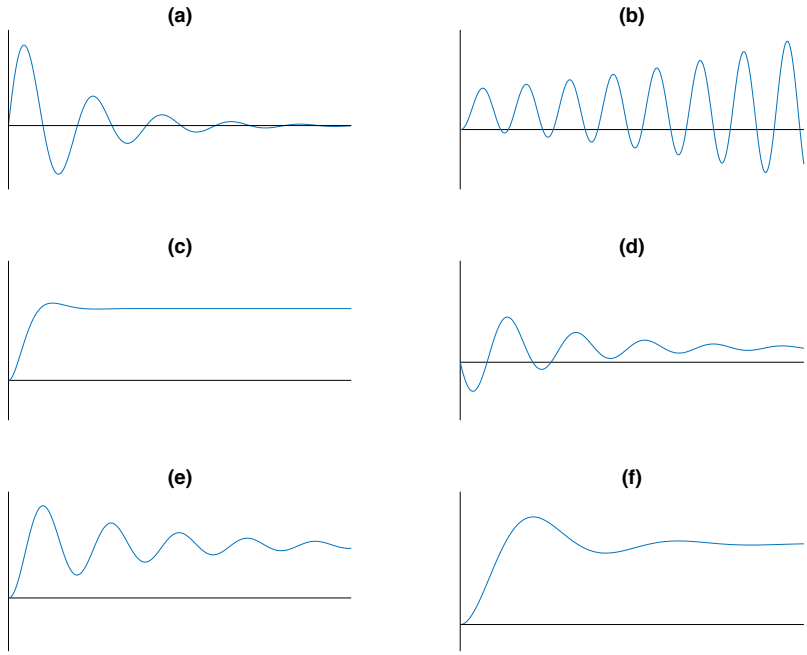
Assume we have the following feedback loop:



where $G(s) = \frac{800}{s^3+80s^2+54s+8}$, and the Bode plot for G(s) is shown on the next page.

(a) If K=1, which of the plots below is a unit step response for $\frac{Y(s)}{X(s)}$? This problem is about "stability margins".) The initial value of the step response is zero. <u>Also, show work to estimate, and label on your chosen plot these 3:</u>

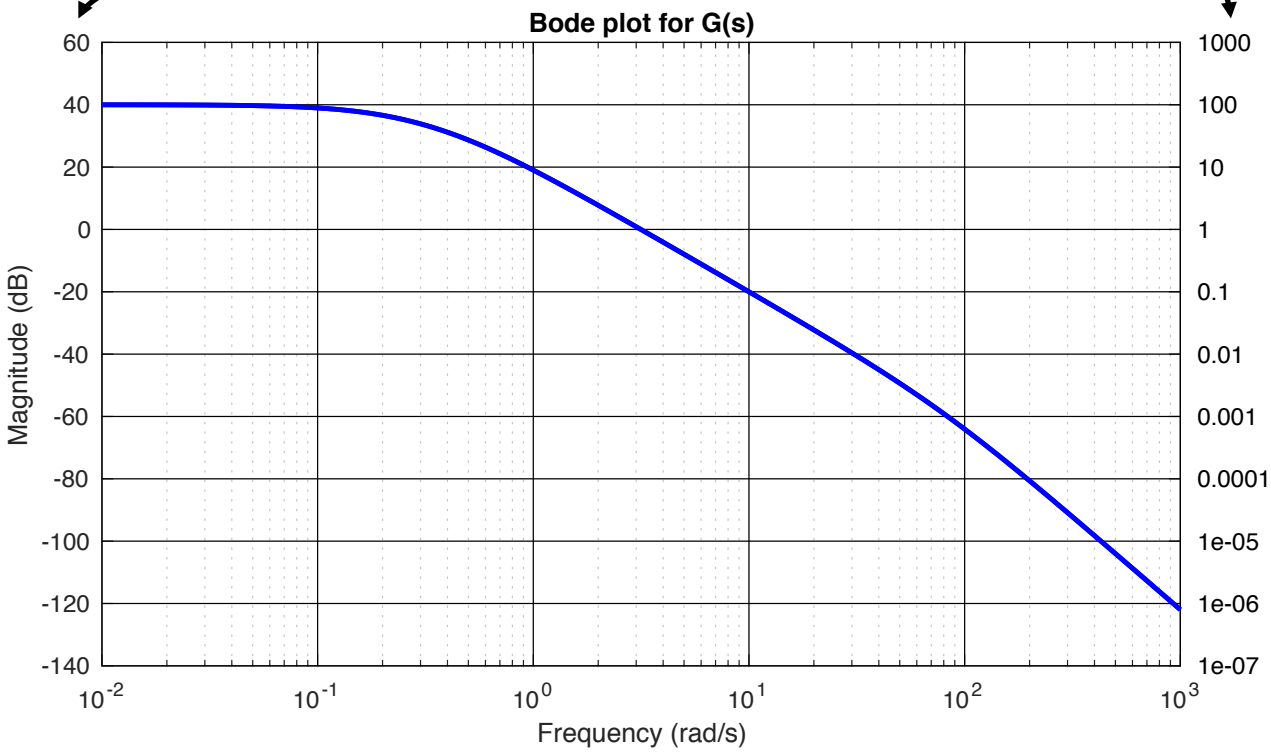
1. t_{peak},
2. the percentage overshoot, and
3. the final value of the step response you choose.

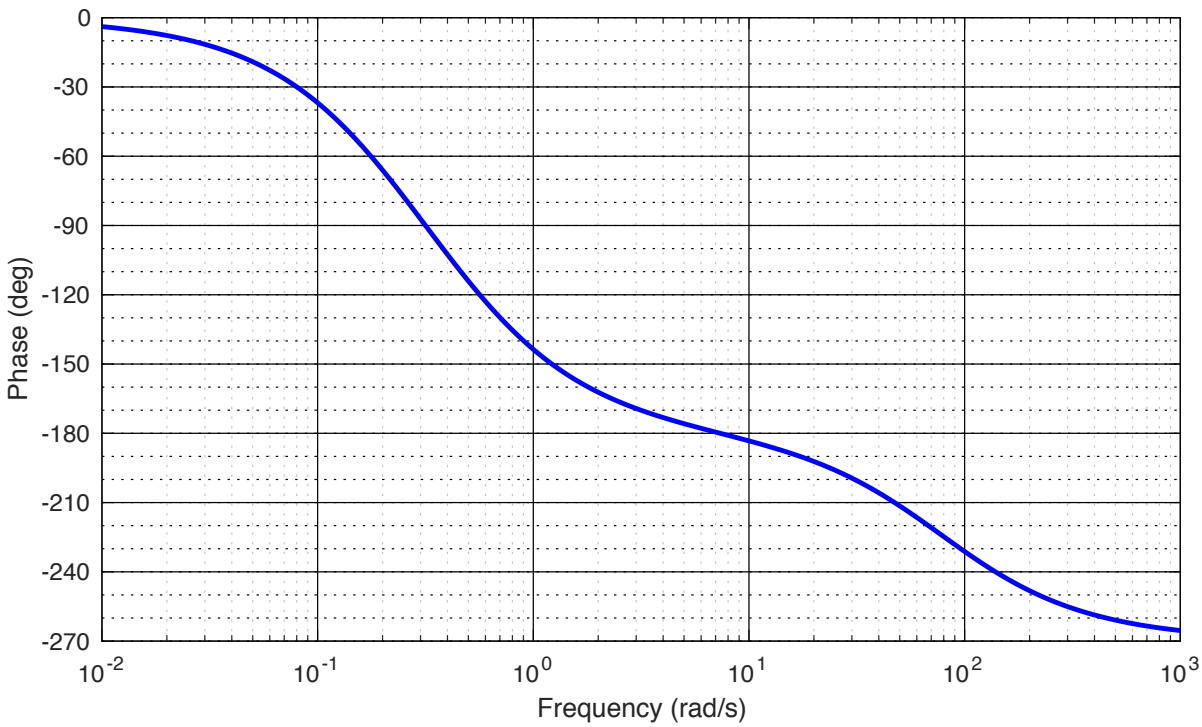


(b) If K=0.1, which plot shows a step response for $\frac{Y(s)}{X(s)}$? Explain briefly. (No labeling needed.) [Hint: magnitude on a Bode plot would be shifted; phase does not change.]

(c) If K=10, which plot shows a step response for $\frac{Y(s)}{X(s)}$? Explain briefly. (No labeling needed.)

Note: Magnitude plot shows both decibels (at left) and actual magnitude (at right)

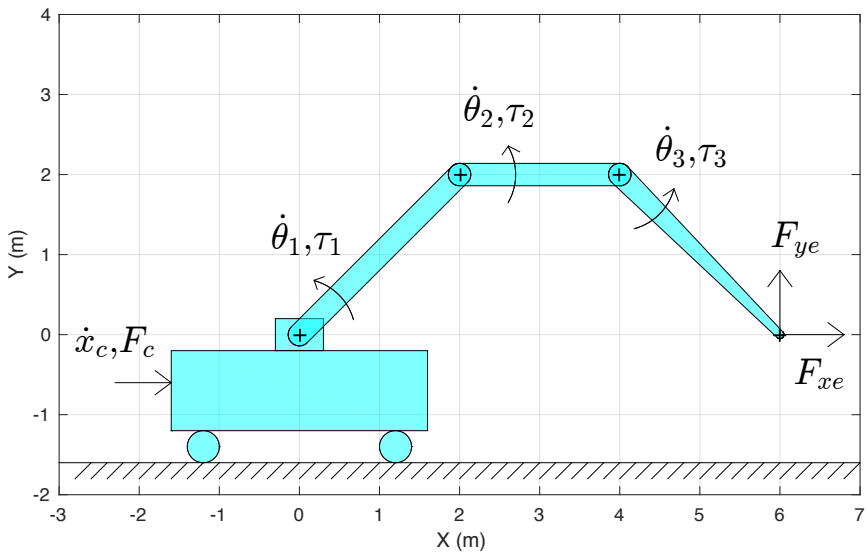




Problem 3 – Non-conservative terms in the Lagrangian equations of motion

You are working on a project with a lunar rover, shown below. We will model this as a cart with a 3-link planar arm mounted on it, as shown below, so that:

$\xi = \begin{bmatrix} x_c \\ \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix}$, where x_c is the x coordinate of the cart and all angles are measured with relative coordinates (i.e., relative to the previous part of the robot this link is connected to).



Since there are 4 DOF's (degrees of freedom), there will be 4 EOM's (equations of motion).

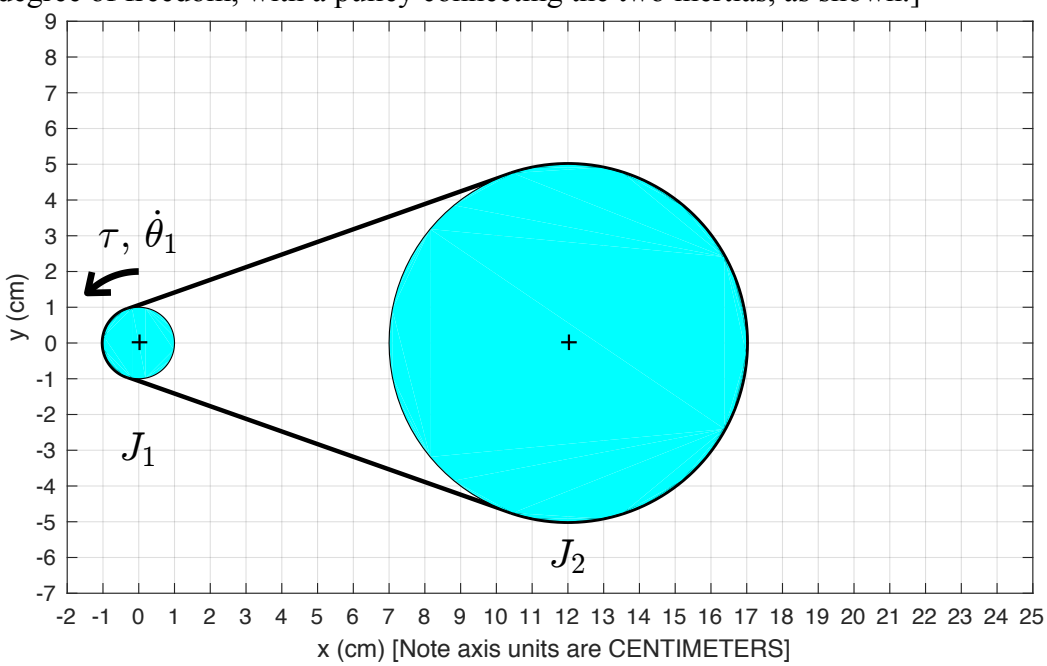
(a) For the configuration shown above, solve for Ξ_1, Ξ_2, Ξ_3, and Ξ_4 (non-conservative forces and/or torques that would go on the righthand side of the equations of motion). Note that the following 6 terms are the only non-conservative forces and torques: F_c (an external force pushing on the cart), τ_1, τ_2, τ_3 (which are motor torques from motors within the robot arm), and F_{xe} and F_{ye} (which are external forces applied at location (6,0), where the end effector is).

Problem 4 – Feedback linearization (and reflected impedance/inertia)

For the system below, $J_1 = 0.01$ (kg-m^2), $J_2 = 0.25$ (kg-m^2), $R_2 = 5R_1$, as drawn, and $\theta_2 = 0$ when $\theta_1 = 0$. Assume you have a desired reference trajectory, $\theta_{ref}(t)$, that you wish to have J_2 (not J_1) follow, and that it is twice differentiable, so that you also have $\dot{\theta}_{ref}(t)$ and $\ddot{\theta}_{ref}(t)$, so that "error" is defined as: $e = \theta_{ref} - \theta_2$.

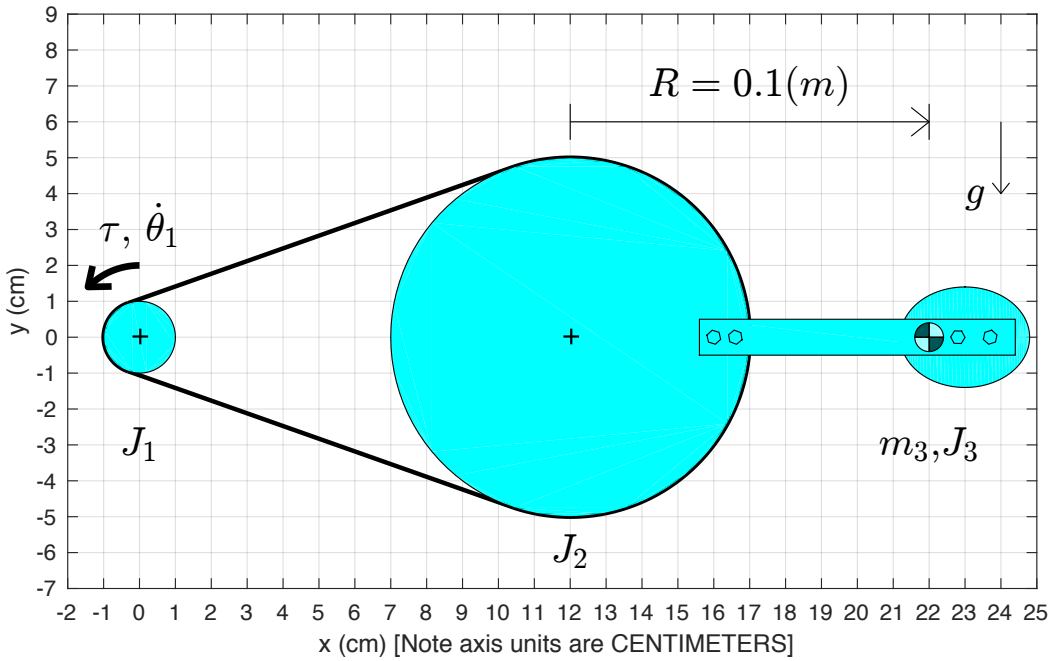
(a) Derive a control law for τ so that the closed-loop system has 2nd-order error dynamics with $\omega_n = 5$ (rad/s) and $\zeta = 0.5$. Note: the torque input, τ, is applied at J_1, while $\theta_{ref}(t)$ is for J_2.

[Hints: Maybe you should derive a Lagrangian EOM, to get started! The system has only one degree of freedom, with a pulley connecting the two inertias, as shown.]



Now, assume we bolt a rigid arm onto the J_2 pulley, as shown. When $\theta_1 = 0$, the system is as shown, with the center of mass of the new "arm" attachment at location, R=0.1(m) from the center of the J_2 pulley, as drawn (note 10 cm = 0.1m; axes are in cm). This new, rigid "arm" has mass m_3=45 (kg) and a moment of inertia about its center of mass of J_3.=0.05 (kg-m^2)

(b) Write the equation of motion for the system below, using θ_1 as the generalized coordinate. Your EOM should look like: $J_{eff}\ddot{\theta}_1$+…= τ. Use the values given to solve for Jeff = _____.



(c) Derive a control law* so that the NEW closed-loop system has second-order dynamics with $\omega_n = 5$ (rad/s) and $\zeta = 0.5$. <u>For this part</u>, just assume $\theta_{ref}(t) = 0$ (meaning its derivatives are ALSO zero). Unlike part (a), your equation will now be non-linear, due to the effects of gravity on the arm part of the system. [*Keeping terms symbolic here is fine.]

END OF EXAM!

Additional Space for Calculations. LABEL PROBLEM(S) YOU ARE WORKING ON!

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