Name________________________________

ECE 179d

Final Exam – Fall 2019

• Extra pages are provided at back of exam. **Turn in all work!**

• Please write ONLY ON THE FRONT sides of exam pages.

• You are allowed two (2) single-sided sheets of notes (or one double-sided sheet)

• No calculators or other computing devices are allowed.

*Good Luck!*

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<thead>
<tr>
<th>Problem</th>
<th>(a)</th>
<th>(b)</th>
<th>(c)</th>
<th>(d)</th>
<th>Total Pts</th>
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P1 _______________
P2 _______________
P3 _______________
P4 _______________

_______________ Total Score

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**Problem 1** – State space equations and linearization

(a) Determine the A and B matrices that correspond to the following dynamic system:

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -20x_1 - 8x_2 + 4u_1
\end{align*}
\]

where \(X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\) and \(U = \begin{bmatrix} u_1 \end{bmatrix}\).

(b) Now, assume we use “state feedback” of the following form: \(U = -KX\). Solve for K such that the closed-loop poles of the system will both be at \(s = -10\). (In other words, we will have two closed-loop poles with \(\omega_n = 10\) (rad/s) and \(\zeta = 1\).) Be careful to get the correct “size” of K, to match your equations in part (a).
(c) Now, linearize a new system, as defined below, to generate A and B matrices, ONLY.
\[
\begin{align*}
y' &= 2y + 2z^2 + 2u \\
z' &= -5y - 2z
\end{align*}
\]
Specifically, linearize the system near \( y^* = -0.8, \ z^* = 2, \ u^* = 3.2 \) (At these values, \( y' = z' = 0 \)). Define a linearized model so that \( U = [u - u^*] \), and \( X = [y - y^* \ z - z^*] \). Note: you are \underline{ONLY} asked to find the A and B matrices here! (Hint: there is only one “nonlinear” term… so most of the terms require no special treatment.)

(d) What are the open-loop poles of the linearized system from part (c)? (If you write the polynomial in \( s \) that determines the poles, you will get most of the credit just for doing this… If you did not solve part (a), then define A and B matrices with open variables (a, b, c, …), to show you know what size they should be, and to be able to proceed with part (d), here.)
Problem 2 – Loop shaping and stability margins

Assume we have the following feedback loop:

\[ X(s) \xrightarrow{+} \sum \xrightarrow{K} G(s) \xrightarrow{} Y(s) \]

where \( G(s) = \frac{800}{s^3 + 80s^2 + 54s + 8} \), and the Bode plot for \( G(s) \) is shown on the next page.

(a) If \( K = 1 \), which of the plots below is a unit step response for \( \frac{Y(s)}{X(s)} \)? This problem is about “stability margins”.) The initial value of the step response is zero. Also, show work to estimate, and label on your chosen plot these 3:
1. \( t_{\text{peak}} \)
2. the percentage overshoot, and
3. the final value of the step response you choose.

(b) If \( K = 0.1 \), which plot shows a step response for \( \frac{Y(s)}{X(s)} \)? Explain briefly. (No labeling needed. [Hint: magnitude on a Bode plot would be shifted; phase does not change.]

(c) If \( K = 10 \), which plot shows a step response for \( \frac{Y(s)}{X(s)} \)? Explain briefly. (No labeling needed.)
Note: Magnitude plot shows both decibels (at left) and actual magnitude (at right).
Problem 3 – Non-conservative terms in the Lagrangian equations of motion

You are working on a project with a lunar rover, shown below. We will model this as a cart with a 3-link planar arm mounted on it, as shown below, so that:

\[
\xi = \begin{bmatrix} x_c \\ \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix}, \text{ where } x_c \text{ is the x coordinate of the cart and all angles are measured with relative coordinates (i.e., relative to the previous part of the robot this link is connected to).}
\]

Since there are 4 DOF’s (degrees of freedom), there will be 4 EOM’s (equations of motion).

(a) For the configuration shown above, solve for \( \Xi_1, \Xi_2, \Xi_3, \) and \( \Xi_4 \) (non-conservative forces and/or torques that would go on the righthand side of the equations of motion). Note that the following 6 terms are the only non-conservative forces and torques: \( F_c \) (an external force pushing on the cart), \( \tau_1, \tau_2, \tau_3 \) (which are motor torques from motors within the robot arm), and \( F_{xe} \) and \( F_{ye} \) (which are external forces applied at location (6,0), where the end effector is).
Problem 4 – Feedback linearization (and reflected impedance/inertia)

For the system below, $J_1 = 0.01 \text{ (kg-m}^2\text{)}, J_2 = 0.25 \text{ (kg-m}^2\text{)}, R_2 = 5R_1$, as drawn, and $\theta_2 = 0$ when $\theta_1 = 0$. Assume you have a desired reference trajectory, $\theta_{\text{ref}}(t)$, that you wish to have $J_2$ (not $J_1$) follow, and that it is twice differentiable, so that you also have $\dot{\theta}_{\text{ref}}(t)$ and $\ddot{\theta}_{\text{ref}}(t)$, so that “error” is defined as: $e = \theta_{\text{ref}} - \theta_2$.

(a) Derive a control law for $\tau$ so that the closed-loop system has 2$^{nd}$-order error dynamics with $\omega_n = 5 \text{ (rad/s)}$ and $\zeta = 0.5$. Note: the torque input, $\tau$, is applied at $J_1$, while $\theta_{\text{ref}}(t)$ is for $J_2$.

[Hints: Maybe you should derive a Lagrangian EOM, to get started! The system has only one degree of freedom, with a pulley connecting the two inertias, as shown.]
Now, assume we bolt a rigid arm onto the J2 pulley, as shown. When θ₁ = 0, the system is as shown, with the center of mass of the new “arm” attachment at location, R=0.1(m) from the center of the J2 pulley, as drawn (note 10 cm = 0.1m; axes are in cm). This new, rigid “arm” has mass \( m_3 = 45 \) (kg) and a moment of inertia about its center of mass of \( J_3 = 0.05 \) (kg-m²)

(b) Write the equation of motion for the system below, using \( \theta_1 \) as the generalized coordinate. Your EOM should look like: \( J_{eff} \ddot{\theta}_1 + \ldots = \tau \). Use the values given to solve for Jeff = ________.

(c) Derive a control law* so that the NEW closed-loop system has second-order dynamics with \( \omega_n = 5 \) (rad/s) and \( \zeta = 0.5 \). For this part, just assume \( \theta_{ref}(t) = 0 \) (meaning its derivatives are ALSO zero). Unlike part (a), your equation will now be non-linear, due to the effects of gravity on the arm part of the system. [*Keeping terms symbolic here is fine.]

END OF EXAM!

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Additional Space for Calculations. LABEL PROBLEM(S) YOU ARE WORKING ON!
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