1.22. What is ω_n for the system illustrated in Figure P1.22?

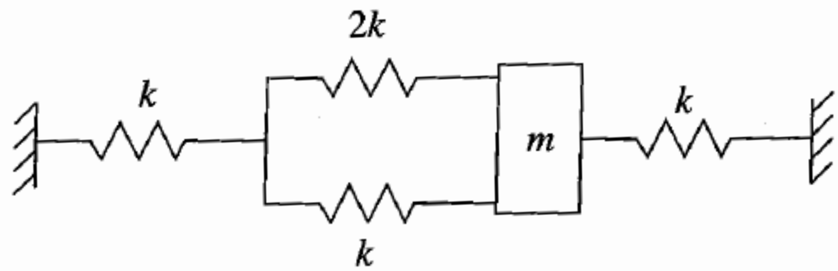


Figure P1.22

1.60. Find the natural frequency about $\theta = 0$ for the system in Figure P1.60. $m_1 = 5$ kg, $m_2 = 5.5$ kg, $l = 1$ m, $g = 9.81$ m/s^2.

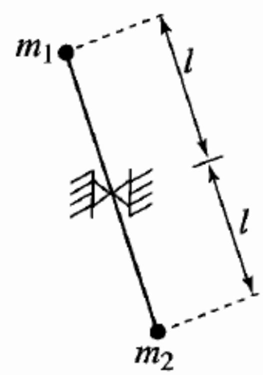


Figure P1.60

1.67. Find the linearized equation of motion about the equilibrium position of the system illustrated in Figure P1.67. You don't have to solve for the equilibrium position itself—just show what you'd need to do to obtain it. The torsional spring is uncompressed when $\theta = 0$. What is the linearized natural frequency?

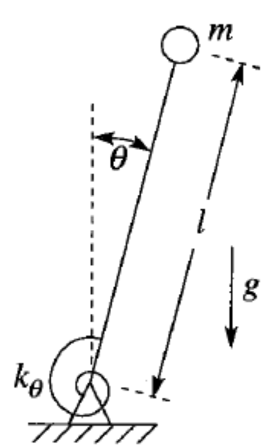


Figure P1.67

1.78. What is the natural frequency for the system shown in Figure P1.78? The hinge at O is frictionless; $k = 10,000$ N/m, $m_1 = 5$ kg, $m_2 = 7$ kg, $\rho = 1$ kg/m, $l_1 = 3$ m, and $l_2 = 5$ m.

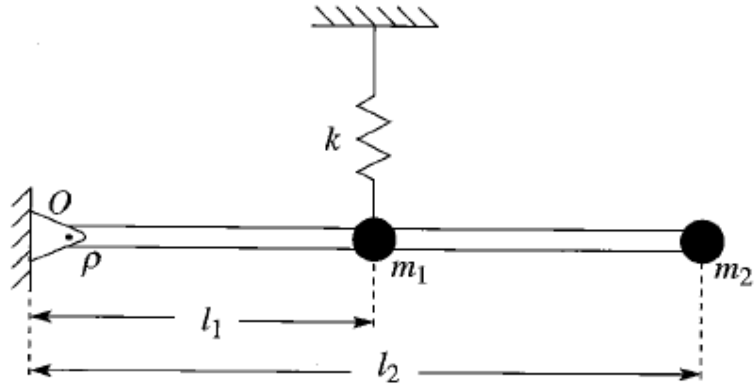


Figure P1.78

1.104. What are the equations of motion for the system illustrated in Figure P1.104?

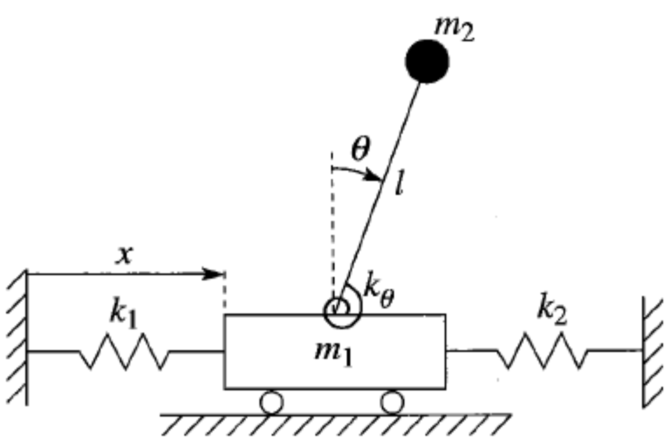


Figure P1.104

2.17. Figure P2.17 shows an enclosure containing a tensioned spring that presses on a mass. The initial compression of the spring is equal to h. The enclosure is subjected to a sinusoidally varying displacement $x(t) = a\cos(\omega t)$. Determine the frequency ω for which the mass will lose contact with the enclosure (consider m, k, h, and a to be fixed, and ignore gravity).

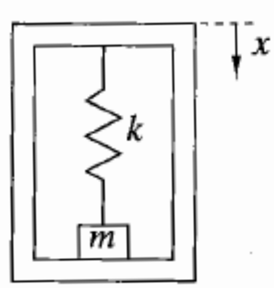


Figure P2.17

2.40. Find the transfer function between the displacement input y and the displacement output x for the system shown in Figure P2.40. $y = \bar{y}\sin(\omega t)$. The rigid bar pivots freely at O.

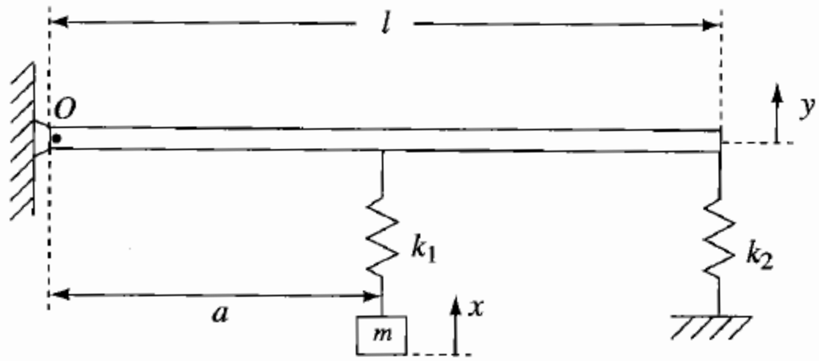


Figure P2.40

2.59. Find the transfer function $\frac{\bar{x}}{\bar{y}}$ for the system illustrated in Figure P2.59. Express the transfer function in both complex and in magnitude/phase form. $m = 10$ kg, $k_1 = 400$ N/m, $c = 14.14$ N·s/m, and $k_2 = 100$ N/m.

2.72. Figure P2.72 illustrates a top view of a top-loading washing machine. The rotational speed of the drum is $\omega = 35$ rad/s and the radius of the drum is $r = .3$ m. Assume that the wet clothes have all gathered into a ball of mass 7 kg (approximated by a point mass). Determine the lateral forces generated by the washing machine. Assume that the mass of the drum is 4 kg.

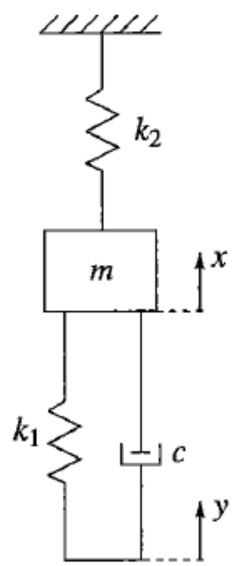


Figure P2.59

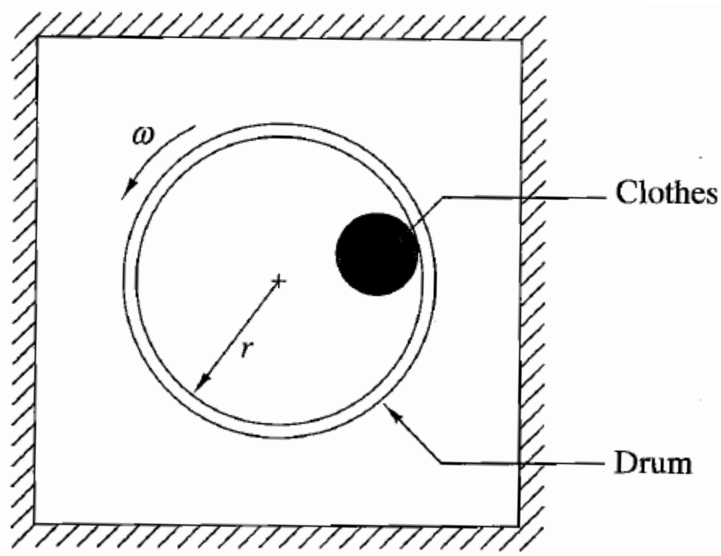


Figure P2.72

3.16. Consider a direct force excited, spring-mass system. Show via a convolution approach that the displacement at $t = .5T$ will be twice the static equilibrium displacement if the force input is constant from $t = 0$ to $t = .5T$.

4.1. Find the two natural frequencies and their associated eigenvectors for the system illustrated in Figure P4.1. $m_1 = 1 \times 10^{-3}$ kg, $m_2 = 10 \times 10^{-3}$ kg, $k_1 = 3 \times 10^3$ N/m, $k_2 = 3 \times 10^3$ N/m.

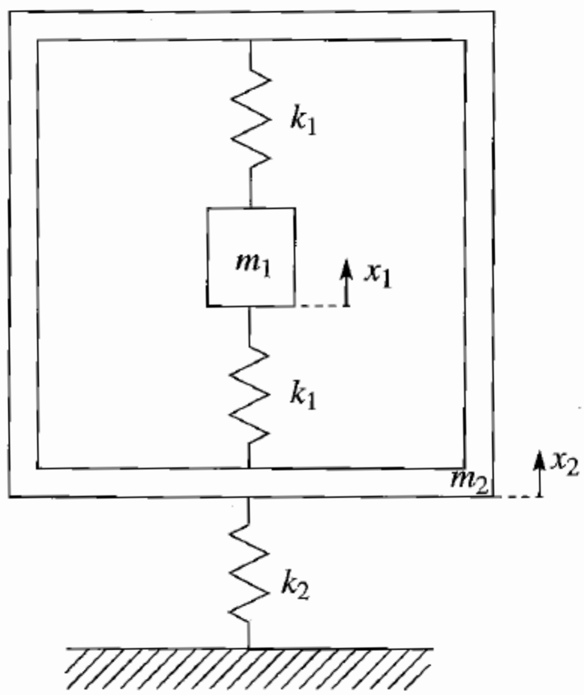


Figure P4.1

4.13. Consider the system illustrated in Figure P4.13. The entire mass is concentrated in three places; the rest of the rigid bar is massless. Find the system's equations of motion. Determine the eigenvectors and natural frequencies. $m_1 = 1$ kg, $m_2 = 1$ kg, $m_3 = 1$ kg, $k = 2$ N/m, $l = 1$ m.

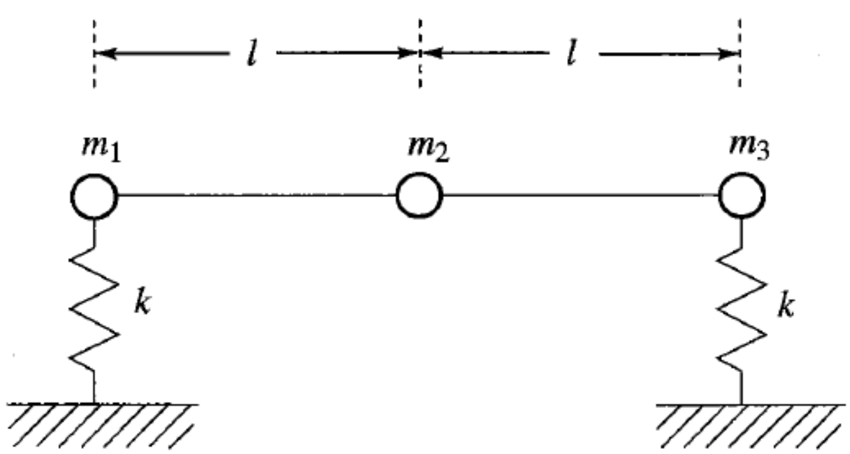


Figure P4.13

4.28. In the system illustrated in Problem 4.28, the three rods have rotational stiffness (k_{θ_1}, k_{θ_2}, and k_{θ_3}) but should be treated as massless (zero rotational inertia). Given the values for k_{θ_i} and I_i, find the system's natural frequencies and associated eigenvectors. $k_{\theta_1} = 8.0 \times 10^5$ N·m, $k_{\theta_2} = 12.0 \times 10^5$ N·m, $k_{\theta_3} = 5.0 \times 10^5$ N·m, $I_1 = 10$ kg·m^2, $I_2 = 30$ kg·m^2.

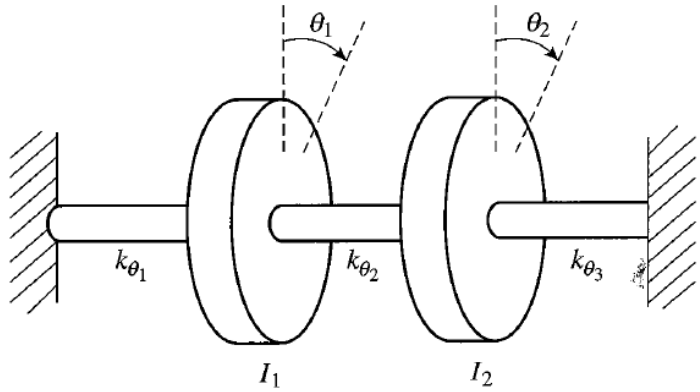


Figure P4.28

4.35. Determine the eigenvalues and eigenvectors of the following system: $a = b = .5$ m, $m_1 = m_2 = 1$ kg, $k = 1$ N/m, $g = 5$ m/s^2 (Obviously the system, shown in Figure P4.35, isn't on Earth.)

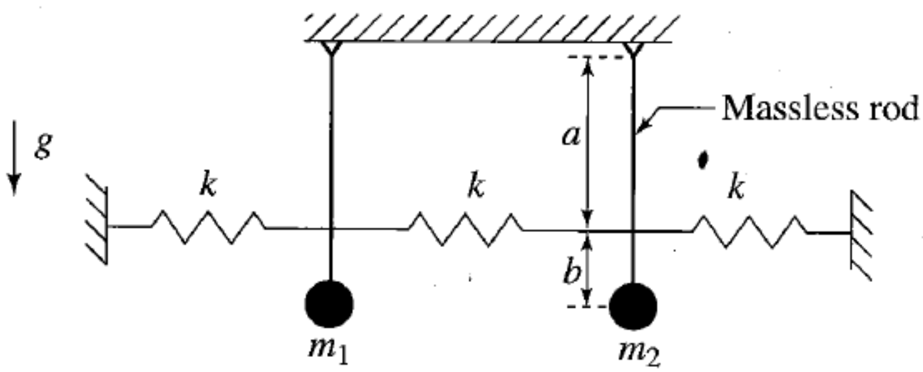


Figure P4.35

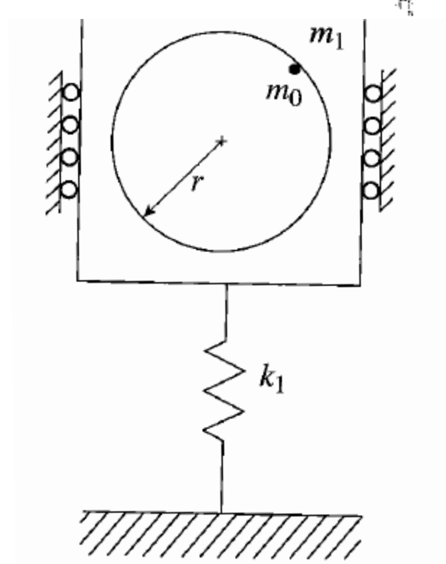
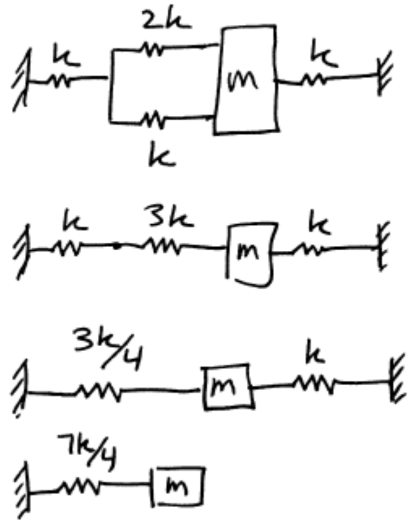


Figure P4.64

4.64. You're asked to analyze the washing machine illustrated in Figure P4.64. The mass m_1 represents the mass of the clothes washer, m_0 represents the largest bunch of clothes likely to be found in the washer, r represents the radius of the washer's drum, and k_1 represents the spring stiffness of the washer's supporting feet. The spin cycle operates at 14 rad/s. Design a spring-mass vibration absorber to be added to the washer so that the amplitude of vibration of the system is less than 3 mm over a range of 4 rad/s centered at the operating frequency. $r = .05$ m, $m_0 = 1$ kg, $m_1 = 9$ kg, $k_1 = 2250$ N/m.

1.22



$$\boxed{\omega_n = \sqrt{\frac{7k}{4m}}}$$

1.60

$$I_0 = l^2 m_2 + l^2 m_1 = 1^2(5+5.5) = 10.5$$

$$I_0 \ddot{\theta} = -m_2 g l \sin\theta + m_1 g l \sin\theta$$

$$I_0 \ddot{\theta} + (m_2 - m_1) g l \sin\theta = 0$$

LINEARIZE: $I_0 \ddot{\theta} + (m_2 - m_1) g l \theta = 0$

$$\boxed{\omega_n = \sqrt{\frac{(m_2 - m_1) g l}{I_0}} = \sqrt{\frac{.5(9.81)(1)}{10.5}} = .683 \text{ RAD/S}}$$

1.67 

Summing moments about the pivot

$$m l^2 \ddot{\theta} = -k_\theta \theta + mgl\sin\theta$$

$$m l^2 \ddot{\theta} + k_\theta \theta - mgl\sin\theta = 0$$

Static Equilibrium: $k_\theta \theta - mgl\sin\theta = 0 \Rightarrow \theta_{eq}$ as solution

Assume small oscillations about equilibrium: $\theta = \theta_{eq} + \eta$ (η small)

Then $\ddot{\theta} = \ddot{\eta}$. Substituting into eq of motion -

$$m l^2 \ddot{\eta} + k_\theta(\theta_{eq} + \eta) - mgl\sin(\theta_{eq} + \eta) = 0$$

$$m l^2 \ddot{\eta} + k_\theta \theta_{eq} + k_\theta \eta - mgl[\sin\theta_{eq}\cos\eta + \cos\theta_{eq}\sin\eta] = 0$$

using $\cos\eta \cong 1$ { $\sin\eta \approx \eta$

$$m l^2 \ddot{\eta} + k_\theta \eta + \overbrace{(k_\theta \theta_{eq} - mgl\sin\theta_{eq})}^{0} - mgl\cos\theta_{eq}\,\eta = 0$$

$$\boxed{m l^2 \ddot{\eta} + (k_\theta - mgl\cos\theta_{eq})\eta = 0}$$

$$\boxed{\omega_n = \sqrt{\frac{k_\theta - mgl\cos\theta_{eq}}{m l^2}}}$$

1.78

$$I_{O_{BAR}} = \frac{1}{3} m l^2 = \frac{1}{3}(5)(5)^2 = 41.\overline{6}$$

$$I_{O_{MASS}} = m_1(3)^2 + m_2(5)^2 = 5(3)^2 + 7(5)^2 = 220$$

$$I_O = I_{O_{BAR}} + I_{O_{MASS}} = 261.\overline{6}$$

$$I_O \ddot{\theta} = -k l_1^2 \sin\theta$$

LINEARIZE: $I_O \ddot{\theta} + k l_1^2 \theta = 0$

$$\boxed{\omega_n = \sqrt{\frac{k l_1^2}{I_O}} = \sqrt{\frac{(10{,}000)9}{261.\overline{6}}} = 18.54 \text{ RAD/S}}$$

1.104 $KE = \frac{1}{2} m_1 \dot{x}^2 + \frac{1}{2} m_2 \left[(\dot{x} + l\dot{\theta} c\theta)^2 + (l\dot{\theta} s\theta)^2 \right]$

$$= \frac{1}{2} m_1 \dot{x}^2 + \frac{1}{2} m_2 \left[\dot{x}^2 + l^2\dot{\theta}^2 + 2l\dot{x}\dot{\theta} c\theta \right]$$

$$PE = \frac{1}{2} k_\theta \theta^2 - m_2 g l (1-\cos\theta) + \frac{1}{2} k_1 x^2 + \frac{1}{2} k_2 x^2$$

$L = KE - PE$. $\frac{\partial L}{\partial \dot{x}} = m_1 \dot{x} + m_2 \dot{x} + m_2 l \dot{\theta} \cos\theta$

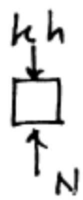
$$\frac{\partial L}{\partial \dot{\theta}} = m_2 (l^2 \dot{\theta} + l \dot{x} \cos\theta)$$

$$\frac{\partial L}{\partial x} = -(k_1 + k_2) x, \quad \frac{\partial L}{\partial \theta} = -m_2 l \dot{x} \dot{\theta} s\theta + m_2 g l \sin\theta - k_\theta \theta$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{x}} - \frac{\partial L}{\partial x} = 0 \Rightarrow \boxed{(m_1 + m_2)\ddot{x} + m_2 l \ddot{\theta} \cos\theta - m_2 l \dot{\theta}^2 \sin\theta + (k_1 + k_2) x = 0}$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}} - \frac{\partial L}{\partial \theta} = 0 \Rightarrow \boxed{l^2 \ddot{\theta} + l \ddot{x} \cos\theta - g l \sin\theta + \frac{k_\theta}{m_2} \theta = 0}$$

2.17



at low ω contact isn't lost and so the mass follows the enclosure's displacement: $x(t)$. Thus we have

$m\ddot{x} = kh - N$ where N is the force of interaction between the mass & enclosure

$-\omega^2 m a \cos\omega t = kh - N$

or $N = kh + \omega^2 m a \cos\omega t$.

Contact is lost when the normal force goes to zero. Since cosine varies from +1 to -1 we have $N = 0$ when $kh = \omega^2 m a$

$$\omega^2 = \frac{kh}{ma} \quad \text{or} \quad \boxed{\omega = \sqrt{\frac{kh}{ma}}}$$

2.40

DISPLACEMENT AT SPRING'S ATTACHMENT POINT IS $y\left(\frac{a}{l}\right)$

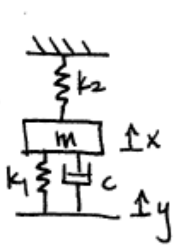
$$m\ddot{x} = -k\left(x - y\left(\frac{a}{l}\right)\right)$$

$$m\ddot{x} + kx = \frac{kay}{l}$$

$$\boxed{\frac{\bar{x}}{\bar{y}} = \frac{ka}{l(k - m\omega^2)}}$$

k_2 DOESN'T AFFECT THE TRANSFER FUNCTION BECAUSE y IS A DISPLACEMENT INPUT.

2.59



$$\omega_n = \sqrt{\frac{500}{10}} = 7.07$$

$$\zeta = \frac{c}{2m\omega_n} = \frac{14.14}{2(10)(7.07)} = 0.1$$

$$m\ddot{x} + c\dot{x} + (k_1 + k_2)x = k_1 y + c\dot{y}$$

Let $x = \bar{x}e^{i\omega t}$, $y = \bar{y}e^{i\omega t}$

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = 40y + 1.414\dot{y}$$

$$[\omega_n^2 - \omega^2 + 2i\zeta\omega\omega_n]\bar{x} = [40 + 1.414i\omega]\bar{y}$$

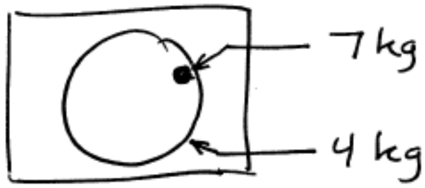
$$\boxed{g(\omega) = \frac{\bar{x}}{\bar{y}} = \frac{(40 + 1.414i\omega)}{\omega_n^2 - \omega^2 + 2i\zeta\omega\omega_n}}$$

$g(\omega) = |g(\omega)|e^{i\phi}$ where

$$\boxed{|g(\omega)| = \sqrt{\frac{1600 + 2\omega^2}{(50 - \omega^2)^2 + 2\omega^2}}}$$

$$\boxed{\phi = \tan^{-1}(0.035\omega) - \tan^{-1}\left(\frac{1.414\omega}{50 - \omega^2}\right)}$$

2.72



$\omega = 35$ RAD/S

ACCELERATION DUE TO CLOTHES: $\omega^2 r = (35)^2(.3) = 367.5$

$\boxed{\text{FORCE MAGNITUDE} = (367.5)(7) = 2572.5 \text{ N}}$

THIS FORCE ACTS AGAINST THE WASHER, ROTATING A FULL CYCLE EVERY $2\pi/\omega = .18$ S.

THE DRUM IS IRRELEVANT TO THE PROBLEM BECAUSE ITS CENTER OF MASS IS STATIONARY

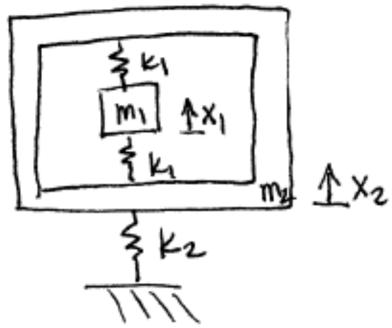
3.16 $m\ddot{x} + kx = f_0$ static displacement: $x_{eq} = f_0/k$

$$h(t) = \frac{1}{m\omega_n}\sin(\omega_n t)$$

$$x\left(\frac{\pi}{\omega_n}\right) = \int_0^{\pi/\omega_n} \frac{1}{m\omega_n}\sin(\omega_n \tau) f_0 \, d\tau = -\frac{f_0}{m\omega_n^2}\left(\cos(\omega_n \tau)\right)\Big|_0^{\pi/\omega_n}$$

$$= \frac{2 f_0}{m\omega_n^2} = \frac{2 f_0}{k}$$

4.1



$m_1 = 1\times10^{-3}$ kg
$m_2 = 10\times10^{-3}$ kg
$k_1 = 3\times10^{3}$ N/m
$k_2 = 1\times10^{3}$ N/m

$$m_1\ddot{x}_1 = -k_1(x_1-x_2) + k_2(x_2-x_1) = -2k_1(x_1-x_2)$$
$$m_2\ddot{x}_2 = -k_2x_2 + k_1(x_1-x_2) + k_1(x_1-x_2) = -k_2x_2 + 2k(x_1-x_2)$$

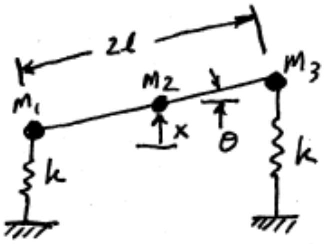
$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} 2k_1 & -2k_1 \\ -2k_1 & 2k_1+k_2 \end{bmatrix}\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$\begin{bmatrix} 1\times10^{-3} & 0 \\ 0 & 10\times10^{-3} \end{bmatrix}\begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} 6\times10^{3} & -6\times10^{3} \\ -6\times10^{3} & 7\times10^{3} \end{bmatrix}\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$\begin{bmatrix} 6\times10^{3} - \omega^2(1\times10^{-3}) & -6\times10^{3} \\ -6\times10^{3} & 7\times10^{3} - \omega^2(10\times10^{-3}) \end{bmatrix}\begin{Bmatrix} \bar{X}_1 \\ \bar{X}_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$\boxed{\begin{array}{ll} \omega_1^2 = 6.609\times10^{6} & \bar{X}_1 = \begin{Bmatrix} 0.9949 \\ -0.1010 \end{Bmatrix} \\ \omega_1 = 2.57\times10^{3}\ \text{rad/s} & \\ \omega_2^2 = 0.09078\times10^{6} & \bar{X}_2 = \begin{Bmatrix} 0.7125 \\ 0.7017 \end{Bmatrix} \\ \omega_2 = 3.01\times10^{2}\ \text{rad/s} & \end{array}}$$

4.13



Use Lagrange's Eq's:

$$KE = \frac{1}{2} m_2 \dot{x}^2 + \frac{1}{2} m_3 (\dot{x} + l\dot{\theta})^2 + \frac{1}{2} m_1 (\dot{x} - l\dot{\theta})^2$$

$$PE = \frac{1}{2} k (x + l\theta)^2 + \frac{1}{2} k (x - l\theta)^2$$

$L = KE - PE$. apply $\frac{d}{dt}\frac{\partial L}{\partial \dot{x}} - \frac{\partial L}{\partial x} = 0$ & $\frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}} - \frac{\partial L}{\partial \theta} = 0$

x: $m_2 \ddot{x} + m_3 (\ddot{x} + l\ddot{\theta}) + m_1 (\ddot{x} - l\ddot{\theta}) + k(x + l\theta) + k(x - l\theta) = 0$

θ: $m_3 (\ddot{x} + l\ddot{\theta}) l + m_1 (\ddot{x} - l\ddot{\theta})(-l) + k(x + l\theta) l - k(x - l\theta) l = 0$

$$\begin{bmatrix} m_1 + m_2 + m_3 & l(m_3 - m_1) \\ l(m_3 - m_1) & (m_3 + m_1) l^2 \end{bmatrix} \begin{Bmatrix} \ddot{x} \\ \ddot{\theta} \end{Bmatrix} + \begin{bmatrix} 2k & 0 \\ 0 & 2kl^2 \end{bmatrix} \begin{Bmatrix} x \\ \theta \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

For our case, all m_i's are the same, leading to a decoupling:

$$\begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix} \begin{Bmatrix} \ddot{x} \\ \ddot{\theta} \end{Bmatrix} + \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} \begin{Bmatrix} x \\ \theta \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

For x motions we have $\boxed{\begin{Bmatrix} X \\ \Theta \end{Bmatrix}_1 = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \text{ \& } \omega_1 = \sqrt{\frac{4}{3}} = 1.155 \text{ rad/s}}$

For θ motions we have $\boxed{\begin{Bmatrix} X \\ \Theta \end{Bmatrix}_2 = \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} \text{ \& } \omega_2 = \sqrt{\frac{2}{1}} = 1.414 \text{ rad/s}}$

4.28 EQ'S OF MOTION

$$I_1\ddot{\theta}_1 = -K_{\theta_1}\theta_1 + K_{\theta_2}(\theta_2-\theta_1)$$

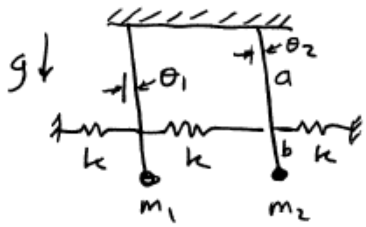
$$I_2\ddot{\theta}_2 = -K_{\theta_2}(\theta_2-\theta_1) - K_{\theta_3}\theta_2$$

$$\begin{bmatrix} I_1 & 0 \\ 0 & I_2 \end{bmatrix}\begin{Bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{Bmatrix} + \begin{bmatrix} K_{\theta_1}+K_{\theta_2} & -K_{\theta_2} \\ -K_{\theta_2} & K_{\theta_2}+K_{\theta_3} \end{bmatrix}\begin{Bmatrix} \theta_1 \\ \theta_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$\begin{bmatrix} 10 & 0 \\ 0 & 30 \end{bmatrix}\begin{Bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{Bmatrix} + \begin{bmatrix} 20\times10^5 & -12\times10^5 \\ -12\times10^5 & 17\times10^5 \end{bmatrix}\begin{Bmatrix} \theta_1 \\ \theta_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$\boxed{\begin{aligned} \omega_1 &= 169.3 \text{ RAD/S}, \quad \Theta = \begin{Bmatrix} .5736 \\ .8191 \end{Bmatrix} \\ \omega_2 &= 477.5 \text{ RAD/S}, \quad \Theta = \begin{Bmatrix} .9738 \\ -.2273 \end{Bmatrix} \end{aligned}}$$

4.35



$a = b = .5$
$m_1 = m_2 = 1$
$k = 1$
$g = 5$

ASSUME SMALL MOTIONS

$$PE = m_1 g(a+b)(1-\cos\theta_1) + m_2 g(a+b)(1-\cos\theta_2) + \tfrac{1}{2}k(a\theta_2 - a\theta_1)^2 + \tfrac{1}{2}k(a\theta_1)^2 + \tfrac{1}{2}k(a\theta_2)^2$$

$$KE = \tfrac{1}{2}m_1(a+b)^2\dot{\theta}_1^2 + \tfrac{1}{2}m_2(a+b)^2\dot{\theta}_2^2$$

$$\theta_1: \quad m_1(a+b)^2\ddot{\theta}_1 + (ka^2 + m_1 g(a+b))\theta_1 - ka^2\theta_2 + ka^2\theta_1 = 0$$

$$\theta_2: \quad m_2(a+b)^2\ddot{\theta}_2 + (ka^2 + m_2 g(a+b))\theta_2 - ka^2\theta_1 + ka^2\theta_2 = 0$$

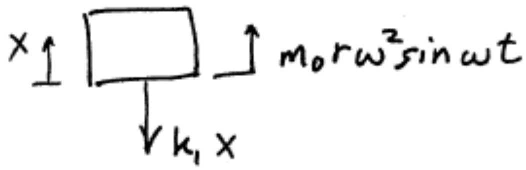
$$\begin{bmatrix} m_1(a+b)^2 & 0 \\ 0 & m_2(a+b)^2 \end{bmatrix}\begin{Bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{Bmatrix} + \begin{bmatrix} 2ka^2 + m_1 g(a+b) & -ka^2 \\ -ka^2 & 2ka^2 + m_2 g(a+b) \end{bmatrix}\begin{Bmatrix} \theta_1 \\ \theta_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{Bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{Bmatrix} + \begin{bmatrix} 5.5 & -.25 \\ -.25 & 5.5 \end{bmatrix}\begin{Bmatrix} \theta_1 \\ \theta_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$\boxed{\begin{aligned} \omega_1 &= 2.2913 \text{ RAD/S}, \quad \underline{X}_1 = \begin{Bmatrix} .7071 \\ .7071 \end{Bmatrix} \\ \omega_2 &= 2.3979 \text{ RAD/S}, \quad \underline{X}_2 = \begin{Bmatrix} -.7071 \\ .7071 \end{Bmatrix} \end{aligned}}$$

4.64 $m_1 = 9\,kg,\; m_0 = 1\,kg,\; \omega = 14\,rad/s,\; k_1 = 2250\,N/m,\; r = .05\,m$

FREE BODY DIAGRAM WITHOUT ABSORBER:



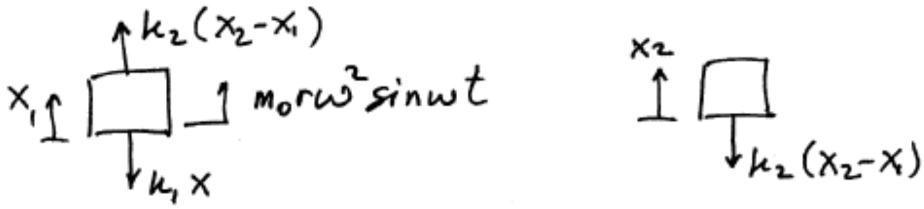
$$(m_0+m_1)\ddot{x} + k_1 x = m_0 r\omega^2 \sin\omega t$$

$$x = \bar{x}\sin\omega t$$

$$[k_1 - \omega^2(m_0+m_1)]\bar{x} = m_0 r\omega^2$$

$$\bar{x} = \frac{m_0 r\omega^2}{k_1 - \omega^2(m_0+m_1)} = .0338\,m$$

FREE BODY DIAGRAM WITH ABSORBER:



$$(m_0+m_1)\ddot{x}_1 + k_1 x_1 - k_2(x_2-x_1) = m_0 r\omega^2 \sin\omega t$$

$$m_2\ddot{x}_2 + k_2(x_2-x_1) = 0$$

using $x_1 = \bar{x}_1 \sin\omega t$

$x_2 = \bar{x}_2 \sin\omega t$

$$-\omega^2\bar{x}_1(m_0+m_1) + k_1\bar{x}_1 - k_2(\bar{x}_2-\bar{x}_1) = m_0 r\omega^2$$

$$-\omega^2\bar{x}_2 m_2 + k_2(\bar{x}_2-\bar{x}_1) = 0$$

$$\bar{x}_1\left(k_1+k_2-\omega^2(m_0+m_1)\right) - k_2\bar{x}_2 = m_0 r\omega^2 \quad (1)$$

$$\bar{x}_2(k_2-\omega^2 m_2) - k_2\bar{x}_1 = 0 \quad (2)$$

$$(2) \Rightarrow \quad \bar{x}_1 = \frac{k_2-\omega^2 m_2}{k_2}\bar{x}_2 \quad 2a)$$

$$(2a) \to (1) \Rightarrow \quad \left(k_1+k_2-\omega^2(m_0+m_1)\right)\left(\frac{k_2-\omega^2 m_2}{k_2}\right)\bar{x}_2 - k_2\bar{x}_2 = m_0 r\omega^2 \quad (1a)$$

$$(1a) \Rightarrow \quad \bar{x}_2 = \frac{k_2 m r\omega^2}{(k_1+k_2-\omega^2(m_0+m_1))(k_2-\omega^2 m_2) - k_2^2} \quad (1b)$$

$$(1b) \to (2a) \Rightarrow \quad \bar{x}_1 = \frac{(k_2-\omega^2 m_2)(m_0 r\omega^2)}{(k_1+k_2-\omega^2(m_0+m_1))(k_2-\omega^2 m_2) - k_2^2}$$

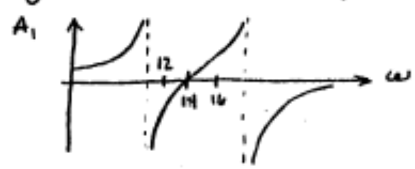
4.64
(CONT)

We want the natural frequency of the vibration absorber to be equal to the operating frequency, so

$$k_2 = \omega^2 m_2 = 196\ m_2$$

Now adjust m_2 to fit the design requirements.

We know the shape of the response curve in the region of the operating frequency,



so we need only calculate the values of X_1 at the endpoints of the frequency range.

With $m_2 = 5.9$ kg, $X_1 = -3$ mm at 12 rad/s and $X_1 = 2.8$ mm at 16 rad/s. This satisfies the design requirements.

Note that m_2 is quite heavy compared to the original system. Larger values for m_2 will also work, and will help to decrease the amplitude of vibration of the absorber.

Note also that now there are two natural frequencies of the combined system. If your operating range is small, this is not a problem, but if you want to increase your range, you may find yourself in trouble again.